Post-graduate lectures: The intergalactic medium

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Introduction

The aim of these notes is to give a brief overview of the physics of the intergalactic medium. For more details, see the recent book by Chris Churchill, for which some chapters are on the web at http://astronomy.nmsu.edu/cwc/Group/Chris/qsonotes.html.

Chapter 1

Scattering of light by the intergalactic medium

Atoms and electrons in the IGM will scatter some of the photons emitted by a distant light source such as a bright quasar. This may make the quasar appear fainter than it really is, or, if the amount scattered is wavelength dependent change the observed spectrum from the emitted spectrum. We need to investigate how light interacts with atoms and electrons to compute these effects.

1.1 The optical depth

Consider a beam of light with intensity I falling onto a plane parallel slab of thickness l, consisting of material with (number) density n. The atoms or molecules in this material may *scatter* some of the infalling light out of the beam, reducing the intensity to $I + dI$, with $dI < 0$. Assuming that the slab scatters a given *fraction* of the light, we can write

$$\frac{dI}{I} = -Andl\,, \tag{1.1}$$

where A is some constant that depends on the material, and has dimension of surface area. When the density is constant, we can integrate this equation to find

$$I = I_0\,\exp(-Anl) = I_0\,\exp(-\tau) \tag{1.2}$$

where the *optical depth*

$$\tau \equiv Anl\,. \tag{1.3}$$

The constant A describes the strength of the interaction between light and a particle in the slab.

1.2 Thomson scattering

A[1] simple example of an interaction between matter and radiation is the scattering of light by a free electron: Thomson scattering. The electric field $\mathbf{E} = \mathbf{E_0}\,\sin(\omega_0 t)$ of the incoming electro-magnetic radiation will exert an oscillating force on the charged particle of charge e and mass m. This will make the particle oscillate and radiate energy. In the classical limit, the fraction of light scattered is independent of frequency, and the cross section is the Thomson cross section:

$$\sigma_T = \frac{8\pi}{3}\,(\frac{e^2}{mc^2})^2\,. \tag{1.4}$$

which has the value $\sigma_T \approx 6.625 \times 10^{-25}\,\mathrm{cm}^2$ for the electron. Note that the interaction strenght $\propto 1/m^2$ so Thomson scattering on protons is much less efficient. Substituting σ_T for A in Eq. (1.3) gives the optical depth for Thomson scattering.

1.3 Scattering by harmonically bound particle

The[2] next simplest system is scattering by a charged particle in a harmonic potential well. The equation of motion of such a system is

$$m\ddot{x} = -m\omega_0^2 x + m\tau\dddot{x} + eE_0\sin(\omega t)\,. \tag{1.5}$$

Here, ω_0 is the frequency of the harmonic oscillator, τ represents a damping term, and the external radiation field is driven with frequency ω. The cross section for scattering in this case is frequency dependent: the particle will radiate much more efficiently when in resonance with the incoming radiation: $\omega \approx \omega_0$,

$$\begin{aligned}
\sigma(\nu) &= \frac{\pi e^2}{mc}\,\frac{\gamma/2\pi}{(\nu-\nu_0)^2+(\gamma/2)^2} \\
&= c\sqrt{\frac{3\pi\sigma_T}{8}}\,\frac{\gamma/2\pi}{(\nu-\nu_0)^2+(\gamma/2)^2} \\
&= \sigma_0\,\phi(\nu) \\
\sigma_0 &= c\sqrt{\frac{3\pi\sigma_T}{8}} = 0.0265\,\mathrm{cm}^2\,\mathrm{s}^{-1}\,.
\end{aligned} \tag{1.6}$$

where $\gamma = \omega_0^2\tau$. Note that $\int_0^\infty \phi(\nu)d\nu = 1$, so that σ_0 is the net cross section of the transition, and ϕ describes the line shape,

$$\phi(\nu) = \frac{\gamma/2\pi}{(\nu-\nu_0)^2+(\gamma/2)^2}\,. \tag{1.7}$$

This shape is called the **Lorentz profile**.

[1] Rybicki & Lightman, section 3.4
[2] Rybicki & Lightman, section 3.6

1.4 Scattering by bound transitions

The[3] interaction of photons with bound states of an atom, for example the electronic states of the Hydrogen atom, can be computed by treating the atom quantum mechanically, but the radiation field classically.

The electron energy levels in the Hydrogen atom have energies $E_n = -E_0/n^2$, where $E_0 = 13.6$ eV is the binding energy. The energy of the photon emitted by a bound-bound transition between levels n_2 and $n_1 < n_2$ is $E_{12} = E_0(1/n_1^2 - 1/n_2^2)$. Alternatively, an atom in energy state n_1 can absorb a photon of energy E_{12} to get into the excited state n_2. The cross section of this process is of course frequency dependent, and is strongly peaked at the frequency $\nu_0 = \nu_{12} = E_{12}/h$, and is given by

$$\sigma(\nu) = \frac{\pi e^2}{mc}\, f\, \frac{\gamma/2\pi}{(\nu - \nu_0)^2 + (\gamma/2)^2}\,. \tag{1.8}$$

The expression is very similar to that of the damped, driven harmonic oscillator, Eq. (1.3). The cross section is reduced by a factor f, imaginatively called the f-value of the transition. The natural line width γ of the transition results from the Heisenberg uncertainty principle.

The f value for a given transition is an integral over the product of the wave-functions of the n_1 and n_2 states, and values for the first three Lyman transitions, (1–2, 1–3, 1–4) are given in Table 2.1. (They can be computed exactly for the Hydrogen atom.)

Because the shape of the line is so similar to that of the driven, damped oscillator, the extended line wings of the Lorentz profile are often called **damping wings** (since for the harmonic oscillator they result from the damping term), and the cosmological structures in which you can detect this wing, are called **damped Lyman-alpha systems**.

1.5 Gunn-Peterson trough

Suppose the IGM were homogeneous, with proper density $n(z)$ (where z is the redshift). What fraction of the light from a distant quasar would be scattered by the hydrogen atoms along the line of sight?

Let $\rho_c = 3H_0^2/(8\pi \mathrm{G})$ be the critical density, and $y \approx 0.24$ be the Helium mass fraction, then the mean number density of hydrogen atoms at redshift $z = 1/a - 1$ is

$$\begin{aligned} n_H(a) &= n_0\, a^{-3} \\ n_0 &= 1.7 \times 10^{-7}\, \frac{\Omega_b h^2}{0.02}\, \mathrm{cm}^{-3}\,. \end{aligned} \tag{1.9}$$

[3] Rybicki & Lightman section 10

Observed and emitted frequencies are related by $\nu_{\rm obs} = \nu_{\rm em}/a$, where $a = 1/(1+z)$ is the expansion factor. For a photon in an FRW metric, the proper lenght $dl = cdt = cda/\dot{a} = cda/aH(a)$, and at redshifts ≥ 1 we can use the Einstein-de Sitter approximation for the Hubble constant, $H(a) \approx H_0\,\sqrt{\Omega_m}\,a^{-3/2}$, where H_0 is Hubble's constant today, and $\Omega_m \approx 0.3$ the matter density. We will also assume that the Lyman-alpha cross section is very peaked, so that $\sigma(\nu) \approx \sigma_0\,\delta(\nu - \nu_0)$, where δ is Dirac's delta function.

Recall that by definition

$$\int_a^b f(x)\,\delta(x - x_0)\,dx = f(x_0)\,, \tag{1.10}$$

when x_0 is in the interval $[a, b]$, and zero if x_0 is outside that interval.

The optical depth at frequency ν, when observing a source at emission redshift $z_{\rm em} = 1/a_{\rm em} - 1$, is then

$$\begin{aligned}
\tau(\nu) &= \int_a^1 \sigma(\nu/a)\,n(a)\,\frac{cda}{a\,H(a)} \\
&= \frac{\sigma_0\,n_0\,c}{H_0\,\sqrt{\Omega_m}}\int_a^1 \delta(\nu/a - \nu_0)\,a^{-5/2}\,da \\
&= \frac{\sigma_0\,n_0\,c}{H_0\,\sqrt{\Omega_m}}\,\frac{a^{-3/2}}{\nu_0} \\
&\approx 13000\,h^{-1}\,\frac{\Omega_b\,h^2}{0.02}\,(1+z)^{3/2}\,.
\end{aligned} \tag{1.11}$$

(Note that $\tau(\nu) = 0$ for $\nu \leq \nu_0/a$.) Therefore the optical depth at redshift $z \sim 3$ is about $\tau(z = 3) \approx 1.5\,\times 10^5$. Clearly the reason we see any flux below the Lyman-alpha emission line is that the Universe is very highly ionized, so that $n_{\rm HI}/n_{\rm H} \approx 1 \times 10^{-5}$.

1.6 line-broadening

1.6.1 Thermal broadening

According to the Boltzmann distribution, the fraction of particles of mass m, in a gas in local thermodynamic equilibrium, with velocity in the $z-$direction in the interval $[v_z, v_z + dv_z]$, is

$$N(v_z) = \frac{1}{\sqrt{2\pi kT/m}}\,\exp(-v_z^2/(2kT/m))\,. \tag{1.12}$$

The scattering cross section of each of these particles with peak at the Doppler shifted frequency $\nu = \nu_0\,(1 + v_z/c)$. Therefore the gas absorbs not just at the resonant frequency ν_0, but the Doppler motion of the atoms broadens the line. The line-profile can be found by realizing that the fraction of photons absorbed at the Doppler shifted frequency ν is just proportional to the fraction of particles with velocity v_z: $\phi(\nu)\, d\nu = N(v_z)\, dv_z$, hence the line-shape

$$\begin{aligned}\phi(\nu) &= N(v_z)\,\frac{dv_z}{d\nu} \\ &= \frac{c}{\nu_0}\,\frac{1}{\sqrt{2\pi kT/m}}\,\exp(-(\nu-\nu_0)^2/(2\nu_0^2 kT/mc^2)\,,\end{aligned} \tag{1.13}$$

which can be rewritten as

$$\begin{aligned}\phi(\nu) &= \frac{1}{\sqrt{\pi\,\Delta\nu_D^2}}\,\exp(-(\nu-\nu_0)^2/\Delta\nu_D^2) \\ \Delta\nu_D &= \frac{\nu_0}{c}\sqrt{\frac{2kT}{m}}\,.\end{aligned} \tag{1.14}$$

1.6.2 Voigt profile

Heizenberg's uncertainty principle caused broadening of spectral lines according to the Lorenz profile, Eq. (1.8). In a gas with thermal motions, each atom moving with velocity v_z will produce an absorption line with a Lorentzian shape. The net profile is then the *convolution* of these two profiles:

$$\phi(\nu) = \int_{-\infty}^{\infty} dv_z\,\frac{1}{\sqrt{2\pi kT/m}}\,\exp(-v_z^2/(2kT/m))\,\frac{\gamma/4\pi^2}{(\nu-\nu_0(1+v_z/c))^2+(\gamma/4\pi)^2}\,. \tag{1.15}$$

Define

$$\begin{aligned}u &\equiv \frac{\nu-\nu_0}{\Delta\nu_D} \\ a &\equiv \frac{\gamma}{4\pi\Delta\nu_D} = \frac{\Delta v_{\mathrm{H}}}{b}\,,\end{aligned} \tag{1.16}$$

where $\Delta v_{\mathrm{H}} = 6.06076\times 10^{-3}\mathrm{km\,s^{-1}}$ is the natural line width, and $b = \sqrt{2kT/m} = c\Delta\nu_D/\nu_0$ is the Doppler width.

The line profile is then

$$\begin{aligned}\phi(\nu) &= \frac{H(a,u)}{\pi^{1/2}\,\Delta\nu_D} \\ H(a,u) &= \frac{a}{\pi}\int_{-\infty}^{\infty}\frac{\exp(-y^2)}{a^2+(u-y)^2}\,dy\,.\end{aligned} \tag{1.17}$$

$H(a,u)$ is called the *Voigt* function, and the line-shape $\phi(\nu)$ is called the Voigt profile. IDL has the Voigt profile built-in.

The width γ of the natural line profile is much narrow than the Doppler width in most cases, and the net line profile is then indistinguishable from a Gaussian. However note that the Voigt profile is dominated by the Lorenzian in the wings.

As an example, consider a slab of gas, with HI column density $N_{\rm HI}$, moving with respect to the observer with velocity v_0. This will produce an absorption line centered at wave length $\lambda = \lambda_0(1 + v_0/c)$, and the amount of absorption at velocity v is

$$\tau(v) = c\sqrt{\frac{3\pi\sigma_T}{8}}\, f\, \frac{N_{\rm HI}}{\pi^{1/2}\,\Delta\nu_D}\, H(a, u + \frac{\nu_0 v}{c\Delta\nu_D})\,. \tag{1.18}$$

The factor $c\sqrt{3\pi\sigma_T/8}\,f = \pi e^2 f/mc$ is the net cross section (cfr Eq. 1.8), it is multiplied by the column density $N_{\rm HI}$, and the Voigt profile is shifted from u to $u + (\nu_0 v/c\Delta\nu_D)$ to take into account the cloud's Doppler shift.

1.7 Multiplets

Triply ionized carbon, C IV, and five-times ionized oxygen, O VI, both have two electrons on the $n = 1$ level, and 1 on a higher energy orbit (in the ground state). This gives them the same electronic structure as (neutral) alkali atoms (such as Li). The lone outer electron, called valence electron, feels a different Coloumb potential from the Hydrogen atom, due to the inner electrons.

The lowest energy $n = 2$ states of such a system, has two levels with slightly different energies, resulting from the spin-orbit coupling of the valence electron. As a consequence, transitions to the ground state are doublets. The doublet nature of these lines makes them easy to spot in spectra. A quick look at a periodic table of the elements will convince you that Si IV will also be a doublet: it simply has a whole set of inner orbit filled in.

Chapter 2

Thermal history of the IGM

2.1 Before reionization

At recombination, the free electron keep the gas temperature tightly coupled to the CMB temperature, $T_{\rm gas} \approx T_{\rm CMB}$. The expanding Universe cools the gas adiabatically. For an adiabatic gas, $p \propto \rho^\gamma$ hence $T \propto \rho^{\gamma-1}$. Since $\rho \propto (1+z)^3$, this gives

$$T \propto (1+z)^{3(\gamma-1)}\,, \tag{2.1}$$

hence $T \propto (1+z)$ for the CMB ($\gamma = 4/3$) and $T \propto (1+z)^2$ for the Hydrogen/Helium IGM ($\gamma = 5/3$). Close to recombination, residual electrons left over from recombination can keep $T_{\rm gas} \approx T_{\rm CMB}$ but below $z \approx 100$ the gas cools faster than the CMB.

2.2 After reionization

We have seen that the IGM is very highly ionized at $z \sim 3$. Suppose therefore that the IGM is bathed in the ionizing flux of a population of sources. The argument that lead to Olbers' paradox suggests that the typical neutral hydrogen photon sees very many sources, therefore we will assume that the flux of this putative UV-background is (nearly) uniform, but redshift dependent. Let $J(\nu, z)$ be this UV-flux.

The photo-ionization rate that results from this flux is

$$\Gamma(z) = \int_{\nu_{\rm th}}^{\infty} \frac{4\pi J(\nu,z)}{h\nu}\,\sigma(\nu)\,d\nu\,, \tag{2.2}$$

The first factor converts energy flux to photon flux, the second factor is the photo-ionization cross section for Hydrogen, and we only integrate over those frequencies for which $h\nu > E_0$, the binding energy for the Hydrogen atom. The rate of change of the neutral density is then (assuming pure hydrogen)

$$\frac{dn_{\rm HI}}{dt} = \alpha n_e \, n_{\rm HII} - \Gamma_e n_e \, n_{\rm HI} - \Gamma \, n_{\rm HI} \,, \tag{2.3}$$

The firsts term is the recombination rate (where the atomic constant α is a function of temperature), the second term is the collisional ionization rate (where an electron collides with a neutral hydrogen atom), and the last term is the photo-ionization rate. Neglecting collisions, and defining the neutral fraction $x \equiv n_{\rm HI}/n_H$, we find that the equilibrium neutral abundance

$$x = \frac{\alpha n_H}{\Gamma} \tag{2.4}$$

when $x \ll 1$ (which is true in the IGM below $z = 6$).

A photon needs more energy than $E_0 = 13.6$ eV to ionized Hydrogen. The excess energy of the photon goes into the kinetic energy of the electron, and this excess energy represent a heating term. The heating rate per ionization is

$$\epsilon(z) = \int_{\nu_{\rm th}}^{\infty} \frac{4\pi J(\nu, z)}{h\nu} \, \sigma(\nu) \, (h\nu - h\nu_{\rm th}) d\nu \,, \tag{2.5}$$

since the excess energy per photo-ionization is $(h\nu - h\nu_{\rm th})$.

The heating rate is therefore

$$\rho \frac{du}{dt} = \epsilon \, n_{HI} \,. \tag{2.6}$$

This shows that the photo-heating rate is

$$\frac{dT}{dt} \propto \frac{\alpha\epsilon}{\Gamma} \, n_H \,. \tag{2.7}$$

Note that this is independent of the amplitude of the UV-background J, since both ϵ and Γ are proportional to J. The heating rate is proportional to density.

The photo-ionization cross section, $\sigma(\nu)$, is sharply peaked around $\nu = \nu_{\rm th}$ falling as ν^{-3} at higher frequencies. This means that most photo-ionizations are by low energy photons, and hence the photo-heating rate is low. However, when the gas is ionized from neutral, then all photons ionize a Hydrogen atom, and the heating rate is now

$$\epsilon(z) = \int_{\nu_{\rm th}}^{\infty} \frac{4\pi J(\nu, z)}{h\nu} \, (h\nu - h\nu_{\rm th}) d\nu \,, \tag{2.8}$$

The consequence is that the gas gets heated very strongly during reionization, with a post-reionization temperature that is only weakly dependent on density. After reionization, the gas cools almost adiabatically, with a small heating from the UV-background.

Gas at lower densities cools more, and gets heated less. This introduces a temperature-density relation that looks like $T \propto \rho^{\gamma-1}$, where $\gamma \approx 1$ immediately after reionization, and γ increasing at lower redshifts.

Appendix I: Atomic constants

Table 2.1: f-values and wavelengths for first three Lyman-series lines in the Hydrogen atom

line name	λ (Angstrom)	f-value
α	1215.6701	0.4164
β	1025.7223	0.0791
γ	972.7431	0.02899

The natural line width for the Hydrogen Lyman-alpha transition,

$$\Delta v_{\mathrm{H}} = 6.06076 \times 10^{-3}\mathrm{km\,s}^{-1}\,, \tag{2.9}$$

(index 'H' to remind you the 'damping' is due to the Heisenberg uncertainty principle) is usually much narrow than the Doppler width ,

$$\Delta v_{\mathrm{D}} = \sqrt{\frac{2kT}{m}} = 12.84\,\mathrm{km\,s}^{-1} = 1.284 \times 10^{6}\,\mathrm{cm\,s}^{-1}\,, \tag{2.10}$$

making the Gaussian line profile usually a good approximation. However for very large optical depths, the Lorenz profile, which is wider in the wings of the line, becomes important, and one needs to use the full Voigt profile shape, Eq. (1.17), of the line. In terms of the parameter a of Eq. (1.16),

$$a = \frac{\Delta v_{\mathrm{H}}}{\Delta v_{\mathrm{D}}}\,. \tag{2.11}$$

An extensive set of atomic constants is part of Carswell's VPFIT programme, and is located at http://www.ast.cam.ac.uk/%7Erfc/atomdat.html.

Appendix II: Home work

1. The binding energy of the hydrogen atom is 13.6 eV. Use this to calculate the wavelengths of the first three Lyman-lines (α, β and γ), and of the first three Balmer lines. [2 marks]

2. Sketch the spectrum of a $z = 3$ QSO, indicating

   (a) the Lyman-alpha forest. Why is the amount of absorption so much less than we expected from the Gunn-Peterson argument? Is there also a Lyman-beta forest? If so, indicate that on your sketch as well. [4 marks]

   (b) Is there a Balmer-alpha forest (which would correspond to the Balmer lines, absorption starting from $n = 2$ instead of $n = 1$ for the Lyman series)? Why (not)? [2 marks]

   (c) Is there a HeI and/or a HeII forest? If so, indicate that on your sketch a well. [2 marks]

3. Assume a spherical cloud of neutral hydrogen has uniform density $n = 1\ \mathrm{cm}^{-3}$ hydrogen atoms, and radius $R = 1$ Mpc. Calculate the Lyman-alpha column density for a line of sight through the centre. [2 marks]

4. Write a programme to evaluate optical depth as function of wavelength for the Voigt profile of Eq. (1.18), produced by a Lyman-alpha cloud at redshift $z = 3$.

   (a) Plot the corresponding absorption line for HI column density $N_{\mathrm{HI}} = 10^{12}, 10^{13}, \cdots, 10^{21}\ \mathrm{cm}^{-2}$ assuming gas temperature $T = 2 \times 10^4$ K. [4 marks]

   (b) Plot the corresponding absorption line for HI column density $N_{\mathrm{HI}} = 10^{14}\ \mathrm{cm}^{-2}$, for temperature $T = 1, 2, 4, 10 \times 10^4$ K. [4 marks]