Cosmology homework

Assume for the questions below a flat ($k = 0$) Universe with cosmological parameters ($\Omega_m = 0.3, \Omega_\Lambda = 0.7, h = 0.72$), and use a radiation density $\Omega_r = 4.22 \times 10^{-5} h^{-2}$ in case you cannot neglect the radiation. If possible, express your answers in a way that is independent of the actual value of the Hubble parameter h.

1. Give the expression for, compute, and plot

   (a) the co-moving distance r

   (b) the angular size-diamater distance d_A

   (c) the luminosity distance d_L

   (d) the age of the Universe t

   for redshifts $z = [0, 10]$. Does d_A reach a maximum? If so, what is its value, and at which redshift does it occur? [10 marks]

2. Show that the Friedmann equation for a spatially flat Universe ($k = 0$) containing matter, no radiation, and a cosmological constant, can be written as

$$\left(\frac{\dot{a}}{a}\right)^2 = t_m^{-2} a^{-3} + t_\Lambda^{-2}\,, \tag{1}$$

   in terms of two characteristic and constant times, t_m and t_Λ. [2 marks] Show that the analytical solution is

$$a(t) = \left(\frac{1}{2}\exp(-3t/2t_\Lambda)\,(\exp(3t/t_\Lambda) - 1)\,(\frac{t_\Lambda}{t_m})\right)^{2/3}\,. \tag{2}$$

   [2 marks]

3. A redshift survey uses the Lyman-α emission line ($\lambda = 1215.67$Å) to detect high-redshift galaxies. The filter is centred on wavelength $\lambda = 4862.24$Å and has width $\Delta\lambda = 30$Å. You may assume the filter is a perfect top-hat. The image size of the telescope used is 5 arc min on a side. Calculate

   (*i*) the co-moving volume surveyed.

   (*ii*) the physical volume surveyed.

These observations detect a particularly interesting galaxy, which is followed-up by more observations. Its observed surface brightness is $\mu = 28$ magnitude per square arc seconds. Calculate its intrinsic surface brightness. [6 marks]